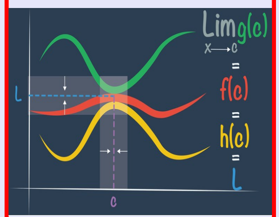
Calculus I

Lecture 2



Feb 19-8:47 AM

Class QZ 1

Use the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

to Solve $2x^2 + 3x - 5 = 0.$

Discriminant

$\boxed{\text{Box Your Answer}}$

$a=2,\ b=3,\ c=-5$

$b^2-4ac = 3^2-4(2)(-5) = \boxed{49}$

$x = \dfrac{-3 \pm \sqrt{49}}{2(2)} = \dfrac{-3 \pm 7}{4}$

$x = \dfrac{-3-7}{4} = \dfrac{-10}{4} = \boxed{\dfrac{-5}{2}}$

$x = \dfrac{-3+7}{4} = \dfrac{4}{4} = \boxed{1}$

$\left\{\dfrac{-5}{2}, 1\right\}$

Jun 16-10:46 AM

Consider $f(x) = 4 - x^2$ Polynomial

1) Domain

$(-\infty, \infty)$

2) y-Int $x = 0$

$f(0) = 4 - 0^2 = \boxed{4}$

$\to (0,4)$

3) x-Int $y = 0$

$f(x) = 0$

$4 - x^2 = 0$

$-x^2 = -4$

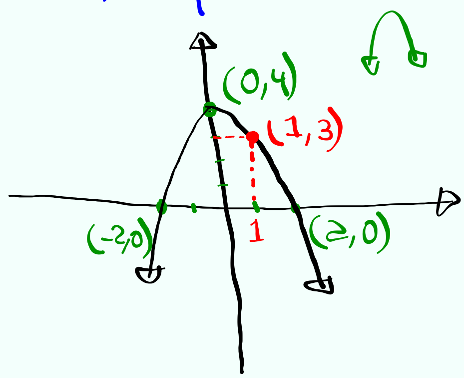
$x^2 = 4$

$x = \pm\sqrt{4}$

$x = \boxed{\pm 2}$

$(2,0)$, $(-2,0)$

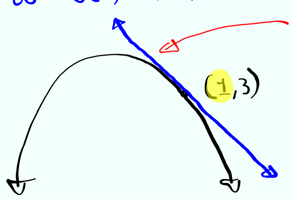
4) Graph Parabola



5) Find $f(1) = 4 - 1^2 = \boxed{3}$

Jun 17-8:11 AM

6) Find slope of the tan. line to the graph of $f(x) = 4 - x^2$ at $x = 1$.



$f(1) = 3$

$m_{\text{tan. line}} = \dfrac{f(x) - f(a)}{x - a}$ if $x = a$

$= \dfrac{4 - x^2 - 3}{x - 1} = \dfrac{1 - x^2}{x - 1}$

$= \dfrac{(1+x)\cancel{(1-x)}^{-1}}{\cancel{x-1}}$

$= -1(1+x)$

when $x = 1$

$m = -1(1+1)$

$= \boxed{-2}$

Recall

$\dfrac{a-b}{a-b} = 1$, $\dfrac{a-b}{b-a} = -1$

7) Find eqn of that tan. line.

$y - y_1 = m(x - x_1)$

$y - 3 = -2(x - 1)$

$y = -2x + 2 + 3$

$\boxed{y = -2x + 5}$

Jun 17-8:17 AM

Consider $f(x) = \dfrac{4}{x+2}$

1) Domain $x+2 \neq 0$

$x \neq -2$

$(-\infty, -2) \cup (-2, \infty)$

2) Y-Int $x = 0$ $f(0) = \dfrac{4}{0+2} = 2$

$(0, 2)$

3) x-Int → None

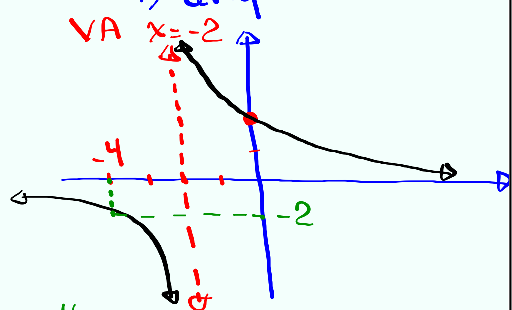
$y = 0$

$f(x) = 0$ $\dfrac{4}{x+2} = 0$

$4 = 0$

No Solution

4) Graph

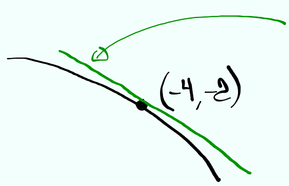


5) Find $f(-4) = \dfrac{4}{-4+2} = \dfrac{4}{-2}$

$= \boxed{-2}$

Jun 17-8:25 AM

6) Find eqn of the tangent line to the graph of $f(x) = \dfrac{4}{x+2}$ at $x = -4$.



$m = \dfrac{f(x) - f(-4)}{x - (-4)}$ when $x = -4$

$= \dfrac{\frac{4}{x+2} - (-2)}{x+4} = \dfrac{\frac{4}{x+2} + 2}{x+4}$

to simplify, use LCD = x+2

$= \dfrac{\cancel{(x+2)} \cdot \frac{4}{\cancel{x+2}} + (x+2)2}{(x+2)(x+4)} = \dfrac{4 + 2x + 4}{(x+2)(x+4)}$

$= \dfrac{2x+8}{(x+4)(x+2)}$ when $x = -4$

$= \dfrac{2\cancel{(x+4)}}{\cancel{(x+4)}(x+2)} = \dfrac{2}{x+2}$

$m = \dfrac{2}{-4+2} = \boxed{-1}$

$y - y_1 = m(x - x_1)$

$y - (-2) = -1(x - -4)$

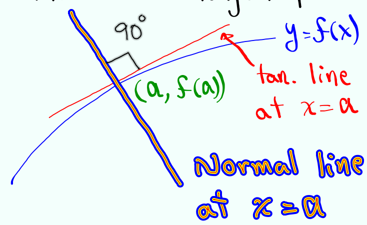
$y + 2 = -x - 4$

$\boxed{y = -x - 6}$

Jun 17-8:34 AM

what is normal line?

Normal line is perpendicular to the tangent line at the tangent point.



$$\boxed{\begin{array}{l}\text{Parallel lines} \Rightarrow m_1 = m_2 \\ \text{Perpendicular lines} \Rightarrow m_1 \cdot m_2 = -1\end{array}}$$

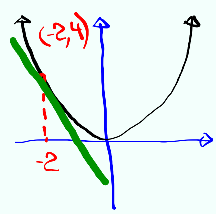
$$m_{\substack{\text{Normal}\\ \text{Line}}} \cdot m_{\substack{\text{tangent}\\ \text{line}}} = -1$$

$$\boxed{m_{\substack{\text{Normal}\\ \text{Line}}} = \frac{-1}{m_{\substack{\text{tan.}\\ \text{line}}}}}$$

Jun 17-8:42 AM

Given $f(x) = x^2$

1) Graph it.



2) find $f(-2)$

$= (-2)^2 = 4$

3) Find $m_{\text{tan. line}}$ at $x = -2$.

$$m_{\text{tan. line}} = \frac{f(x) - f(-2)}{x - (-2)} = \frac{x^2 - 4}{x + 2} = \frac{\cancel{(x+2)}(x-2)}{\cancel{x+2}}$$

when $x = -2$ $\quad = x - 2$

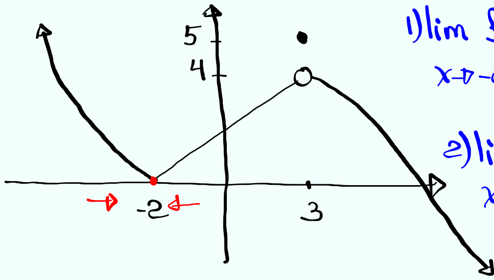
4) Find $m_{\substack{\text{Normal}\\ \text{Line}}}$ at $x = -2$.

$$m_{\substack{\text{Tan.}\\ \text{line}}} = -2 - 2 = \boxed{-4}$$

$$m_{\substack{\text{Normal}\\ \text{Line}}} = \frac{-1}{m_{\substack{\text{Tan.}\\ \text{Line}}}} = \frac{-1}{-4} = \boxed{\frac{1}{4}}$$

Jun 17-8:47 AM

Consider the graph of $f(x)$ below



1) $\lim_{x\to -2^-} f(x) = \boxed{0}$

2) $\lim_{x\to -2^+} f(x) = \boxed{0}$

3) $\lim_{x\to -2} f(x) = \boxed{0}$

4) find $f(-2) = \boxed{0}$

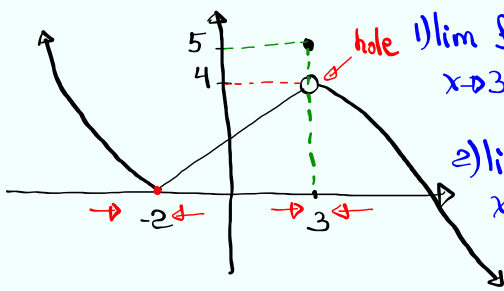
If $\lim_{x\to a} f(x) = f(a)$, then $f(x)$ is continuous at $x=a$.

$f(x)$ is cont. at $x=-2$.

why? $\lim_{x\to -2} f(x) = f(-2)$

Jun 17-8:57 AM

Consider the graph of $f(x)$ below



1) $\lim_{x\to 3^-} f(x) = 4$

2) $\lim_{x\to 3^+} f(x) = 4$

3) $\lim_{x\to 3} f(x) = 4$

4) find $f(3) = 5$

If $\lim_{x\to a} f(x) = f(a)$, then $f(x)$ is continuous at $x=a$.

Is $f(x)$ cont. at $x=3$? NO

why? $\lim_{x\to 3} f(x) \neq f(3)$

Jun 17-8:57 AM

Evaluate

1) $\lim_{x \to 4} (\sqrt{x} + x^2)$

$= \sqrt{4} + 4^2 = 2 + 16 = \boxed{18}$

2) $\lim_{x \to \frac{\pi}{2}} \sin x$ $\quad 90^\circ$

$= \sin \frac{\pi}{2} = \sin 90^\circ = \boxed{1}$

3) $\lim_{x \to \frac{\pi}{4}} (\tan x - 1)$

$= \tan \frac{\pi}{4} - 1$

$= \tan 45^\circ - 1 = 1 - 1 = \boxed{0}$

4) $\lim_{x \to 1} \dfrac{x^2+5x+6}{x+1}$

$= \dfrac{1^2+5(1)+6}{1+1}$

$= \dfrac{12}{2} = \boxed{6}$

Jun 17-9:09 AM

5) $\lim_{x \to -2} \dfrac{x^2+5x+6}{x+2} = \dfrac{(-2)^2+5(-2)+6}{-2+2} = \dfrac{4-10+6}{0} = \dfrac{0}{0}$ I.F.

$= \lim_{x \to -2} \dfrac{\cancel{(x+2)}(x+3)}{\cancel{x+2}} = \lim_{x \to -2} (x+3) = -2+3 = \boxed{1}$

6) $\lim_{x \to 100} \dfrac{\sqrt{x}-10}{x-100} = \dfrac{\sqrt{100}-10}{100-100} = \dfrac{10-10}{100-100} = \dfrac{0}{0}$ I.F.

$= \lim_{x \to 100} \dfrac{\overset{A-B}{(\sqrt{x}-10)}\overset{A+B}{(\sqrt{x}+10)}}{(x-100)(\sqrt{x}+10)} = \lim_{x \to 100} \dfrac{\overset{A^2-B^2}{\cancel{x-100}}^{\,1}}{\cancel{(x-100)}_{1}(\sqrt{x}+10)}$

$= \lim_{x \to 100} \dfrac{1}{(\sqrt{x}+10)}$

$= \dfrac{1}{\sqrt{100}+10}$

$= \dfrac{1}{10+10}$

$= \boxed{\dfrac{1}{20}}$

Jun 17-9:18 AM

7) $\lim_{h\to 0} \dfrac{(3+h)^{-1}-3^{-1}}{h} = \dfrac{(3+0)^{-1}-3^{-1}}{0}$

$= \dfrac{3^{-1}-3^{-1}}{0} = \dfrac{0}{0}$ I.F.

Recall
$x^{-n} = \dfrac{1}{x^n}$

$\lim_{h\to 0} \dfrac{(3+h)^{-1}-3^{-1}}{h} = \lim_{h\to 0} \dfrac{\frac{1}{3+h}-\frac{1}{3}}{h}$

LCD $= (3+h)3$

$= \lim_{h\to 0} \dfrac{\cancel{(3+h)}\cdot 3\cdot \frac{1}{\cancel{3+h}} - (3+h)\cdot\cancel{3}\cdot\frac{1}{\cancel{3}}}{(3+h)\cdot 3\cdot h}$

$= \lim_{h\to 0} \dfrac{3 - (3+h)\cdot 1}{(3+h)\cdot 3\cdot h} = \lim_{h\to 0} \dfrac{\cancel{3}-\cancel{3}-\overset{-1}{\cancel{h}}}{(3+h)\cdot 3\cdot \cancel{h}}$

$= \lim_{h\to 0} \dfrac{-1}{(3+h)\cdot 3} = \dfrac{-1}{(3+0)\cdot 3} = \boxed{\dfrac{-1}{9}}$

Jun 17-9:28 AM

Limit laws:

Assume these limits exist.

1) $\lim_{x\to a} c = c$

2) $\lim_{x\to a} cf(x) = c\lim_{x\to a} f(x)$

3) $\lim_{x\to a} [f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

4) $\lim_{x\to a} [f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$

5) $\lim_{x\to a} [f(x)\cdot g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$

6) $\lim_{x\to a} \left[\dfrac{f(x)}{g(x)}\right] = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x) \neq 0$

Jun 17-9:53 AM

Suppose $\lim_{x \to a} f(x) = 5$, $\lim_{x \to a} g(x) = -4$

Find

1) $\lim_{x \to a} 4f(x) = 4 \lim_{x \to a} f(x) = 4 \cdot 5 = \boxed{20}$

2) $\lim_{x \to a} [f(x) \cdot g(x) + 20]$

$= \lim_{x \to a} [f(x) \cdot g(x)] + \lim_{x \to a} 20$

$= \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) + 20$

$= 5 \cdot (-4) + 20 = \boxed{0}$

3) $\lim_{x \to a} \frac{f(x) - 2}{g(x) + 3} = \frac{\lim_{x \to a} [f(x) - 2]}{\lim_{x \to a} [g(x) + 3]} = \frac{\lim_{x \to a} f(x) - \lim_{x \to a} 2}{\lim_{x \to a} g(x) + \lim_{x \to a} 3}$

$= \frac{5 - 2}{-4 + 3} = \frac{3}{-1}$

$= \boxed{-3}$

Jun 17-10:00 AM

Find $\lim_{x \to a} f(x)$ & $\lim_{x \to a} g(x)$ if

$$\begin{cases} \lim_{x \to a} [f(x) + 2g(x)] = 5 \\ \lim_{x \to a} [f(x) - g(x)] = -10 \end{cases} \Rightarrow \begin{cases} \lim_{x \to a} f(x) + 2 \lim_{x \to a} g(x) = 5 \\ \lim_{x \to a} f(x) - \lim_{x \to a} g(x) = -10 \end{cases}$$

Solve

$$\begin{cases} A + 2B = 5 \\ A - B = -10 \end{cases} \cdot (-1) \Rightarrow \begin{cases} A + 2B = 5 \\ -A + B = 10 \end{cases}$$

$3B = 15$

$A - 5 = -10$

$\boxed{B = 5}$

$\boxed{A = -5}$

$\lim_{x \to a} f(x) = -5$

$\lim_{x \to a} g(x) = 5$

Jun 17-10:10 AM

More laws

7) $\lim_{x\to a} [f(x)]^n = [\lim_{x\to a} f(x)]^n$

8) $\lim_{x\to a} x^n = a^n$

9) $\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$

Be aware of even roots & radicand.

10) $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$

Jun 17-10:16 AM

Evaluate if $\lim_{x\to 2} f(x) = -3$

1) $\lim_{x\to 2} [x^2 - f(x)] = \lim_{x\to 2} x^2 - \lim_{x\to 2} f(x)$

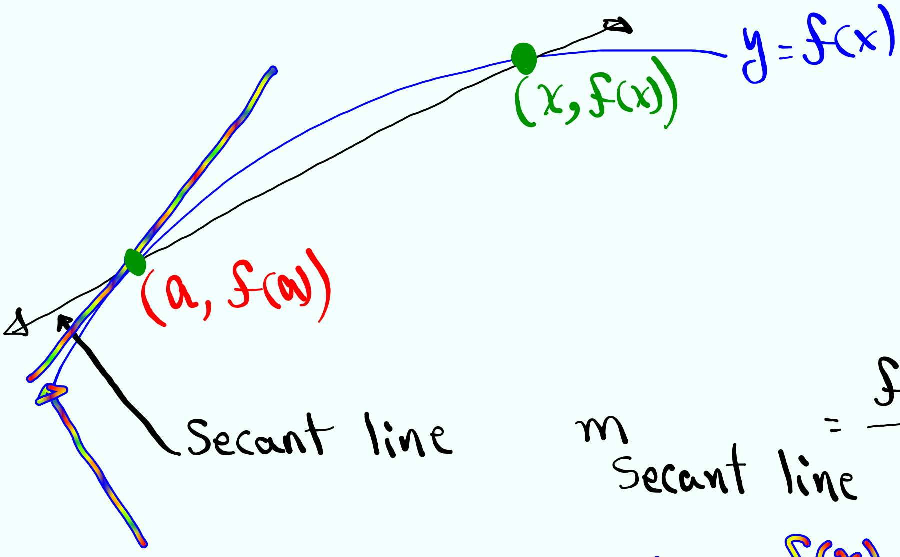
$= 2^2 - (-3) = \boxed{7}$

2) $\lim_{x\to 2} [x^3 f(x) + 24] = \lim_{x\to 2} x^3 \cdot \lim_{x\to 2} f(x) + 24$

$= 2^3 \cdot (-3) + 24$

$= -24 + 24 = \boxed{0}$

Jun 17-10:20 AM

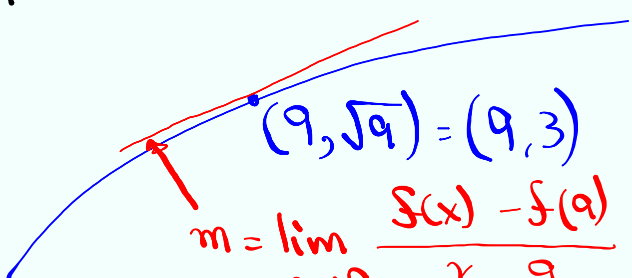


$$m_{\text{Secant line}} = \frac{f(x)-f(a)}{x-a}$$

$$m_{\text{tan. line}} = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

Jun 17-10:26 AM

Find slope of the tan. line to the graph $f(x) = \sqrt{x}$ at $x = 9$.



$$m = \lim_{x \to 9} \frac{f(x) - f(9)}{x - 9} = \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$$

$$\frac{\sqrt{9}-3}{9-9} = \frac{0}{0} \quad \text{I.F.}$$

$$= \lim_{x \to 9} \frac{(\sqrt{x}-3)\cancel{(\sqrt{x}+3)}}{\cancel{(x-9)}(\sqrt{x}+3)} = \lim_{x \to 9} \frac{1}{\sqrt{x}+3} = \boxed{\frac{1}{6}}$$

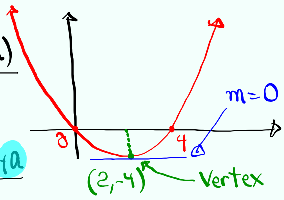
Jun 17-10:29 AM

Find all points where $f(x) = x^2 - 4x$ has horizontal tangent line.

Parabola
opens up
Y-Int $(0,0)$
x-Ints $(0,0), (4,0)$

$$m_{\text{tan. line}} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{x^2 - 4x - (a^2 - 4a)}{x - a}$$



$$= \lim_{x \to a} \frac{x^2 - 4x - a^2 + 4a}{x - a}$$

$$= \lim_{x \to a} \frac{\boxed{x^2 - a^2}\ \boxed{-4x + 4a}}{x - a} = \lim_{x \to a} \frac{(x-a)(x+a) - 4(x-a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cancel{(x-a)}(x+a-4)}{\cancel{x-a}} = \lim_{x \to a} (x + a - 4)$$

$$= a + a - 4 = 2a - 4$$

we want $m = 0$ $\quad 2a - 4 = 0 \to \boxed{a = 2}$

Point $(2, f(2)) = (2, -4)$

Jun 17-10:34 AM

Squeeze Thrm:

If $f(x) \le g(x) \le h(x)$ and

$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$ then

$\lim_{x \to a} g(x) = L$

Suppose $x^3 + 1 \le f(x) \le x^2 - 1$

Find $\lim_{x \to -1} f(x)$

$\lim_{x \to -1} (x^3 + 1) = (-1)^3 + 1 = 0$

$\lim_{x \to -1} (x^2 - 1) = (-1)^2 - 1 = 0$

by S.T.,

$\lim_{x \to -1} f(x) = \boxed{0}$

Jun 17-10:47 AM